Measurement schemes for quantum field theory in curved spacetimes

CJ Fewster
University of York

First Virtual LQP/AQFTUK
June 2020

arXiv:1810.06512 - with Rainer Verch; short summary in arXiv:1904.06944
arXiv:2003.04660 with Henning Bostelmann and Maximilian Ruep

A gap in the literature

Measurement theory in quantum mechanics has a long and controversial history.

- Simple rules are taught to students
- Measurement chain analysed in quantum measurement theory

A gap in the literature

Measurement theory in quantum mechanics has a long and controversial history.

- Simple rules are taught to students
- Measurement chain analysed in quantum measurement theory

Much less is said in quantum field theory

- Lecture courses and texts are silent
- QMT rarely discussed for QFT; still less in curved spacetimes.
- Algebraic QFT is founded on the idea of local observables,
  but little discussion of how they are actually measured.

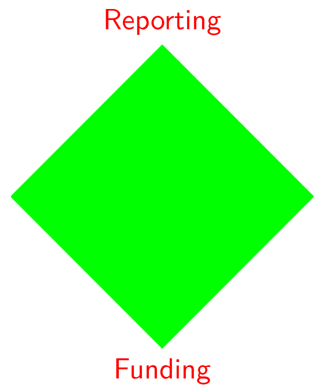
In this talk...

- Analyse the measurement chain in QFT
- Provide a general operational framework for measurement
- Covariant; applies in curved as well as flat spacetime
- Passes consistency tests
- Can be used for calculation

Relativity, quantum theory and measurement – it's complicated

$c < \infty$

measurements occupy bounded spacetime regions

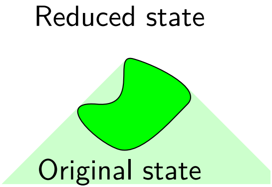


Relativity, quantum theory and measurement – it's complicated

$c < \infty$	measurements occupy bounded spacetime regions	
$\hbar > 0$	...but are not performed at points	$\Delta E \Delta t \gtrsim \hbar$

Relativity, quantum theory and measurement – it's complicated

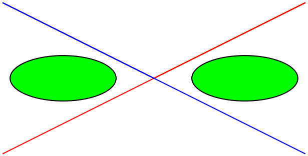
$c < \infty$	measurements occupy bounded spacetime regions
$\hbar > 0$	...but are not performed at points
Lorentz invariance	no preferred frame no instantaneous collapse at constant t



Hellwig & Kraus

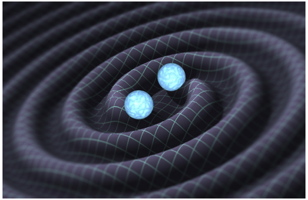
Relativity, quantum theory and measurement – it's complicated

$c < \infty$	measurements occupy bounded spacetime regions
$\hbar > 0$	...but are not performed at points
Lorentz invariance	no preferred frame no instantaneous collapse at constant t
Relativity of simultaneity	no preferred order for spacelike separated measurements



Relativity, quantum theory and measurement – it's complicated

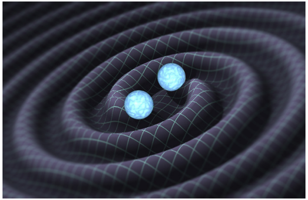
$c < \infty$	measurements occupy bounded spacetime regions
$\hbar > 0$	...but are not performed at points
Lorentz invariance	no preferred frame no instantaneous collapse at constant t
Relativity of simultaneity	no preferred order for spacelike separated measurements
Curved spacetime	lack of symmetry, nontrivial topology...



Ligo

Relativity, quantum theory and measurement – it's complicated

$c < \infty$	measurements occupy bounded spacetime regions
$\hbar > 0$	...but are not performed at points
Lorentz invariance	no preferred frame no instantaneous collapse at constant t
Relativity of simultaneity	no preferred order for spacelike separated measurements
Curved spacetime	lack of symmetry, nontrivial topology...

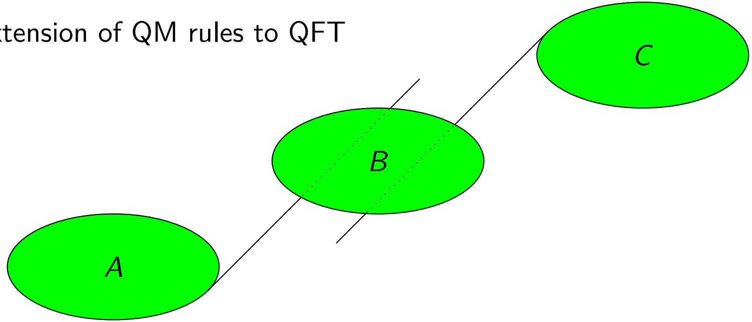


Ligo

Extension of QM measurement rules to QFT is nontrivial and risks pathology

Impossible measurements Sorkin 1993

'By hand' extension of QM rules to QFT

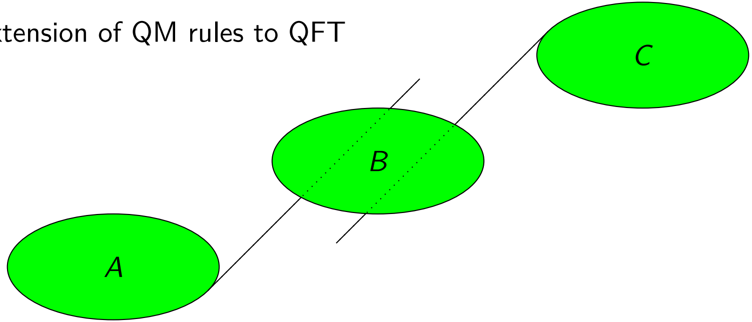


Claim: nonselective measurement of a typical observable B allows C to determine whether A has conducted a measurement – superluminal communication.

Presumably, therefore, B represents an impossible measurement.

Impossible measurements Sorkin 1993

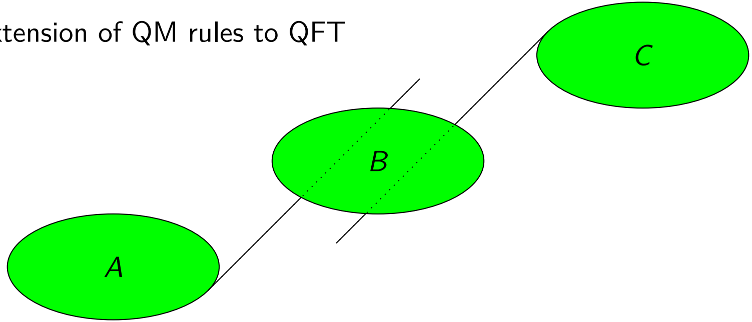
'By hand' extension of QM rules to QFT



Spacetime extension of B is critical.

Impossible measurements Sorkin 1993

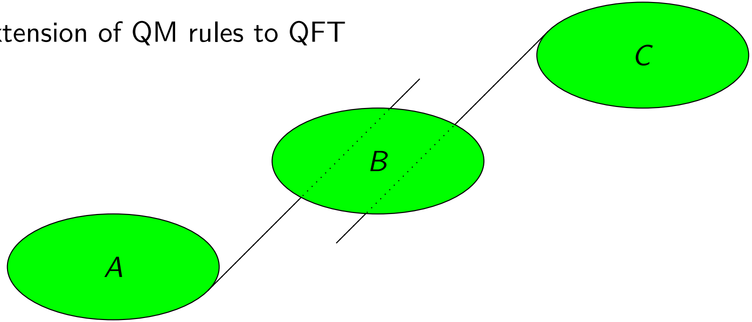
'By hand' extension of QM rules to QFT



"[I]t becomes a priori unclear, for quantum field theory, which observables can be measured consistently with causality and which can't.

Impossible measurements Sorkin 1993

'By hand' extension of QM rules to QFT



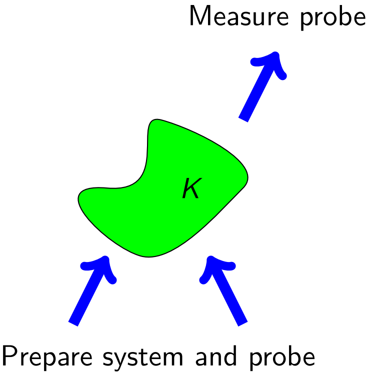
"[I]t becomes a priori unclear, for quantum field theory, which observables can be measured consistently with causality and which can't.
This would seem to deprive [QFT] of any definite measurement theory, leaving the issue of what can actually be measured to (at best) a case-by-case analysis"
See e.g., Borsten, Jubb, Kells (2019) for such an analysis.

Operational approach CJF & Verch, 2018

Instead of constructing rules for QFT *de novo*, apply a systematic approach by modelling the measurement process.

Operational approach CJF & Verch, 2018

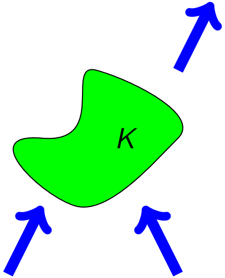
A QFT (system) is coupled to another QFT (probe) in a compact spacetime region K (a proxy for the experimental design). The probe is measured elsewhere.



(a pr

Measurements are performed on the coupled system–probe set-up, but are described in the language of a fictitious uncoupled system.

Measure probe



Prepare system and probe

$\overset{?}{\approx}$

Updated state

Hypothetical local system measurement

Prepare system

Outline of the idea

Describe the system and probe by QFTs $\mathcal{A}$, $\mathcal{B}$ on spacetime $\boldsymbol{M}$ (globally hyperbolic). $\mathcal{A}(\boldsymbol{M})$ is the algebra of system observables on $\boldsymbol{M}$. We compare

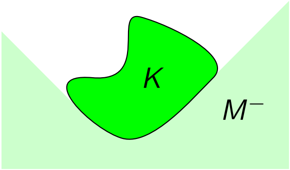
- the uncoupled combination $\mathcal{U}$ of $\mathcal{A}$ and $\mathcal{B}$
- a coupled combination $\mathcal{C}$ with compact coupling region K.

Outline of the idea

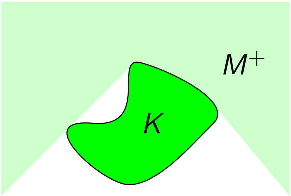
Describe the system and probe by QFTs $\mathcal{A}$, $\mathcal{B}$ on spacetime $\boldsymbol{M}$ (globally hyperbolic). $\mathcal{A}(\boldsymbol{M})$ is the algebra of system observables on $\boldsymbol{M}$. We compare

- the uncoupled combination $\mathcal{U}$ of $\mathcal{A}$ and $\mathcal{B}$
- a coupled combination $\mathcal{C}$ with compact coupling region K.

Define geometrical 'in'/'out' regions $M^{-/+}$ on which $\mathcal{U}$ and $\mathcal{C}$ agree.



$$M^{\pm} = M \setminus J^{\mp}(K)$$



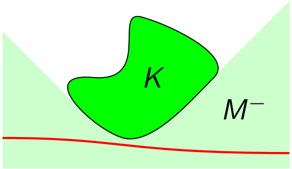
Outline of the idea

Describe the system and probe by QFTs $\mathcal{A}$, $\mathcal{B}$ on spacetime $\boldsymbol{M}$ (globally hyperbolic). $\mathcal{A}(\boldsymbol{M})$ is the algebra of system observables on $\boldsymbol{M}$. We compare

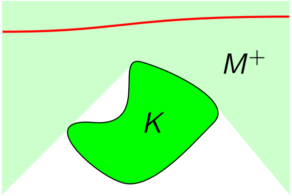
- the uncoupled combination $\mathcal{U}$ of $\mathcal{A}$ and $\mathcal{B}$
- a coupled combination $\mathcal{C}$ with compact coupling region K.

Define geometrical 'in'/'out' regions $M^{-/+}$ on which $\mathcal{U}$ and $\mathcal{C}$ agree.

$M^{\pm}$ contain Cauchy surfaces $\implies$ $\exists$ identifications $\tau^{\pm} : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$.



$$M^{\pm} = M \setminus J^{\mp}(K)$$



Outline of the idea

Describe the system and probe by QFTs $\mathcal{A}$, $\mathcal{B}$ on spacetime $\boldsymbol{M}$ (globally hyperbolic). $\mathcal{A}(\boldsymbol{M})$ is the algebra of system observables on $\boldsymbol{M}$. We compare

- the uncoupled combination $\mathcal{U}$ of $\mathcal{A}$ and $\mathcal{B}$
- a coupled combination $\mathcal{C}$ with compact coupling region K.

Define geometrical 'in'/'out' regions $M^{-/+}$ on which $\mathcal{U}$ and $\mathcal{C}$ agree.

$M^{\pm}$ contain Cauchy surfaces $\implies \exists$ identifications $\tau^{\pm} : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$.

$\tau^{\pm}$ translate fictitious uncoupled language to the physical coupled system.

- 'Prepare system and probe states separately at early times'
- 'Measure a probe observable at late times'

Measurements on the probe are interpreted as measurements of system observables.

Algebraic QFT

Describe a QFT on $\boldsymbol{M}$ in terms of a $*$-algebra $\mathcal{A}(\boldsymbol{M})$ with unit and subalgebras $\mathcal{A}(\boldsymbol{M}; N)$ for suitable open regions $N \subset \boldsymbol{M}$.

Minimal conditions

- **Isotony** $N_1 \subset N_2 \implies \mathcal{A}(\boldsymbol{M}; N_1) \subset \mathcal{A}(\boldsymbol{M}; N_2)$
- **Timeslice** $\mathcal{A}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M})$ if N contains a Cauchy surface of $\boldsymbol{M}$
- **Einstein** $[\mathcal{A}(\boldsymbol{M}; N_1), \mathcal{A}(\boldsymbol{M}; N_2)] = 0$ if $N_{1,2}$ are causally disjoint

Algebraic QFT

Describe a QFT on $\boldsymbol{M}$ in terms of a $*$-algebra $\mathcal{A}(\boldsymbol{M})$ with unit and subalgebras $\mathcal{A}(\boldsymbol{M}; N)$ for suitable open regions $N \subset \boldsymbol{M}$.

Minimal conditions

Isotony $N_1 \subset N_2 \implies \mathcal{A}(\boldsymbol{M}; N_1) \subset \mathcal{A}(\boldsymbol{M}; N_2)$

Timeslice $\mathcal{A}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M})$ if N contains a Cauchy surface of $\boldsymbol{M}$

Einstein $[\mathcal{A}(\boldsymbol{M}; N_1), \mathcal{A}(\boldsymbol{M}; N_2)] = 0$ if $N_{1,2}$ are causally disjoint

$A = A^* \in \mathcal{A}(\boldsymbol{M}; N)$ is interpreted by fiat as an observable localisable in N

NB An observable may be localisable in many distinct regions.

Algebraic QFT

Describe a QFT on $\boldsymbol{M}$ in terms of a $*$-algebra $\mathcal{A}(\boldsymbol{M})$ with unit and subalgebras $\mathcal{A}(\boldsymbol{M}; N)$ for suitable open regions $N \subset \boldsymbol{M}$.

Minimal conditions

Isotony $N_1 \subset N_2 \implies \mathcal{A}(\boldsymbol{M}; N_1) \subset \mathcal{A}(\boldsymbol{M}; N_2)$

Timeslice $\mathcal{A}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M})$ if N contains a Cauchy surface of $\boldsymbol{M}$

Einstein $[\mathcal{A}(\boldsymbol{M}; N_1), \mathcal{A}(\boldsymbol{M}; N_2)] = 0$ if $N_{1,2}$ are causally disjoint

$A = A^* \in \mathcal{A}(\boldsymbol{M}; N)$ is interpreted by fiat as an observable localisable in N
NB An observable may be localisable in many distinct regions.

A state is a positive, normalised linear functional $\omega : \mathcal{A}(\boldsymbol{M}) \to \mathbb{C}$, assigning an expectation value $\omega(A)$ to $A \in \mathcal{A}(\boldsymbol{M})$.

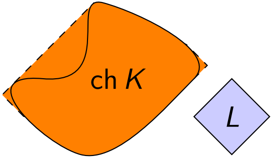
Coupled combinations and scattering

Describe both the system and the probe by AQFTs $\mathcal{A}$, $\mathcal{B}$ on $\boldsymbol{M}$.

Their uncoupled combination is $\mathcal{U} = \mathcal{A} \otimes \mathcal{B}$.

Theory $\mathcal{C}$ is a coupled combination of $\mathcal{A}$ and $\mathcal{B}$ with compact coupling region K.

$$\mathrm{ch}\,(K) = J^+(K) \cap J^-(K)$$



Minimal abstract definition:
$\forall L$ outside the causal hull $\mathrm{ch}\,(K)$
$\exists$ an isomorphism

$$\mathcal{U}(\boldsymbol{M}; L) \to \mathcal{C}(\boldsymbol{M}; L)$$

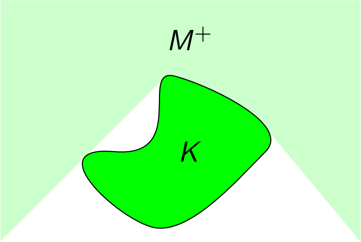
compatible with isotony.

Coupled combinations and scattering

Describe both the system and the probe by AQFTs $\mathcal{A}$, $\mathcal{B}$ on $\boldsymbol{M}$.

Their uncoupled combination is $\mathcal{U} = \mathcal{A} \otimes \mathcal{B}$.

Theory $\mathcal{C}$ is a coupled combination of $\mathcal{A}$ and $\mathcal{B}$ with compact coupling region K.



$M^\pm = M \setminus J^\mp(K)$ contain Cauchy surfaces for $\boldsymbol{M}$

Applying timeslice, $\exists$ isomorphisms

$$\tau^\pm : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$$

$$\tau^\pm : \mathcal{U}(\boldsymbol{M}) = \mathcal{U}(\boldsymbol{M}; M^\pm) \longrightarrow \mathcal{C}(\boldsymbol{M}; M^\pm) = \mathcal{C}(\boldsymbol{M})$$

Coupled combinations and scattering

Describe both the system and the probe by AQFTs $\mathcal{A}$, $\mathcal{B}$ on $\boldsymbol{M}$.

Their uncoupled combination is $\mathcal{U} = \mathcal{A} \otimes \mathcal{B}$.

Theory $\mathcal{C}$ is a coupled combination of $\mathcal{A}$ and $\mathcal{B}$ with compact coupling region K.

Upshot: covariantly described advanced/retarded response maps

$$\tau^{-/+} : \mathcal{U}(\boldsymbol{M}) \xrightarrow{\cong} \mathcal{C}(\boldsymbol{M})$$

are identifications of the uncoupled and coupled combinations at early/late times. The scattering map is

$$\Theta = (\tau^-)^{-1} \circ \tau^+ \in \mathrm{Aut}(\mathcal{U}(\boldsymbol{M}))$$

Locality: $\Theta \restriction \mathcal{U}(\boldsymbol{M}; N) = \mathrm{id}$, if $N \subset K^\perp$.

Measurement scheme: prepare early, measure late

Describe measurements of $\mathcal{C}(\boldsymbol{M})$ in uncoupled language.

Fixing a probe preparation state σ and system state ω, the state

$$\underset{\sim}{\omega}_\sigma = ((\tau^-)^{-1})^*(\omega \otimes \sigma) \qquad \underset{\sim}{\omega}_\sigma(X) = (\omega \otimes \sigma)((\tau^-)^{-1}X)$$

of $\mathcal{C}(\boldsymbol{M})$ is uncorrelated at early times.

Measurement scheme: prepare early, measure late

Describe measurements of $\mathcal{C}(\boldsymbol{M})$ in uncoupled language.

Fixing a probe preparation state σ and system state ω, the state

$$\underset{\sim}{\omega}_\sigma = ((\tau^-)^{-1})^*(\omega \otimes \sigma) \qquad \underset{\sim}{\omega}_\sigma(X) = (\omega \otimes \sigma)((\tau^-)^{-1}X)$$

of $\mathcal{C}(\boldsymbol{M})$ is uncorrelated at early times.

An observable $\widetilde{B} := \tau^+(\mathbf{1} \otimes B)$ tests probe degrees of freedom at late times.

Measurement scheme: prepare early, measure late

Describe measurements of $\mathcal{C}(\boldsymbol{M})$ in uncoupled language.

Fixing a probe preparation state σ and system state ω, the state

$$\underset{\sim}{\omega}_\sigma = ((\tau^-)^{-1})^*(\omega \otimes \sigma) \qquad \underset{\sim}{\omega}_\sigma(X) = (\omega \otimes \sigma)((\tau^-)^{-1}X)$$

of $\mathcal{C}(\boldsymbol{M})$ is uncorrelated at early times.

An observable $\widetilde{B} := \tau^+(\mathbf{1} \otimes B)$ tests probe degrees of freedom at late times.

Measure $\widetilde{B}$ in state $\underset{\sim}{\omega}_\sigma$; interpret as a measurement of a system observable A in state ω chosen so that

$$\omega(A) = \underset{\sim}{\omega}_\sigma(\widetilde{B}) = (\omega \otimes \sigma)(\Theta(\mathbf{1} \otimes B)) \qquad \text{for all } \omega.$$

Measurement scheme - ctd

Problem: find A so that $\omega(A) = (\omega \otimes \sigma)(\Theta(\mathbf{1} \otimes B))$ for all ω

Observation:

$$(\omega \otimes \sigma)(P \otimes Q) = \omega(P)\sigma(Q) = \omega(\sigma(Q)P) = \omega(\eta_\sigma(P \otimes Q))$$

where $\eta_\sigma : \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ linearly extends $P \otimes Q \mapsto \sigma(Q)P$.

Solution:

$$A = \varepsilon_\sigma(B) \stackrel{\text{def}}{=} \eta_\sigma(\Theta(\mathbf{1} \otimes B))$$

Measurement scheme - ctd

Problem: find A so that $\omega(A) = (\omega \otimes \sigma)(\Theta(\mathbf{1} \otimes B))$ for all ω

Observation:

$$(\omega \otimes \sigma)(P \otimes Q) = \omega(P)\sigma(Q) = \omega(\sigma(Q)P) = \omega(\eta_\sigma(P \otimes Q))$$

where $\eta_\sigma : \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ linearly extends $P \otimes Q \mapsto \sigma(Q)P$.

Solution:

$$A = \varepsilon_\sigma(B) \stackrel{\text{def}}{=} \eta_\sigma(\Theta(\mathbf{1} \otimes B))$$

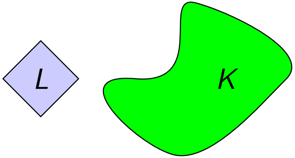
$\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B))$ is called the induced system observable corresponding to probe observable B.

In QMT language, $(\mathcal{C}, \tau^{\pm}, \sigma, B)$ is a measurement scheme for system observable $\varepsilon_\sigma(B)$.

Induced system observables – localisation

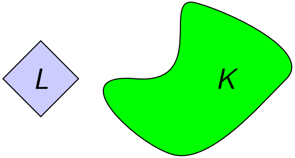
Recall: Θ acts trivially on $\mathcal{U}(\boldsymbol{M}; L)$ if $L \subset K^\perp$.

Theorem (a) If $B \in \mathcal{B}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then $\varepsilon_\sigma(B) = \sigma(B)\mathbf{1}$.



Induced system observables – localisation

Recall: Θ acts trivially on $\mathcal{U}(\boldsymbol{M}; L)$ if $L \subset K^\perp$.

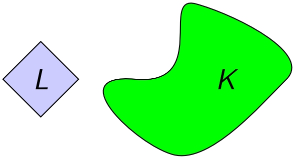


Theorem (a) If $B \in \mathcal{B}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then $\varepsilon_\sigma(B) = \sigma(B)\mathbf{1}$.

(b) If $A \in \mathcal{A}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then $[A, \varepsilon_\sigma(B)] = 0$ for all B.

Induced system observables – localisation

Recall: Θ acts trivially on $\mathcal{U}(\boldsymbol{M}; L)$ if $L \subset K^\perp$.



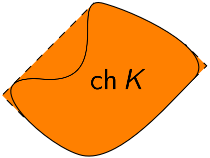
Theorem (a) If $B \in \mathcal{B}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then $\varepsilon_\sigma(B) = \sigma(B)\mathbf{1}$.

(b) If $A \in \mathcal{A}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then $[A, \varepsilon_\sigma(B)] = 0$ for all B.

Corollary If $\mathcal{A}$ obeys a Haag property, then

$$\varepsilon_\sigma(B) \in \mathcal{A}(\boldsymbol{M}; N) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}),$$

where N is any open connected causally convex set containing K.



NB N must contain $\operatorname{ch} K$. The localisation of B is irrelevant. Consistent with the idea that $\mathcal{A}(\boldsymbol{M}; N)$ consists of observables that are measurable in N.

Induced system observables – fluctuations

True and hypothetical expectation values agree, by construction

$$\underset{\sim}{\omega}_\sigma(\widetilde{B}) = \omega(\varepsilon_\sigma(B)) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}).$$

$\varepsilon_\sigma : \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ is linear, completely positive, and obeys

$$\varepsilon_\sigma(\mathbf{1}) = \mathbf{1}, \qquad \varepsilon_\sigma(B^*) = \varepsilon_\sigma(B)^*, \qquad \varepsilon_\sigma(B)^*\varepsilon_\sigma(B) \leq \varepsilon_\sigma(B^*B).$$

Consequently, the true measurement displays greater variance than the hypothetical one due to detector fluctuations

$$\mathrm{Var}(\widetilde{B}; \underset{\sim}{\omega}_\sigma) \geq \mathrm{Var}(\varepsilon_\sigma(B); \omega).$$

Effects and effect-valued measures

An effect is an observable s.t. B and $\mathbf{1} - B$ are positive, corresponding to a true/false measurement

$$\mathrm{Prob}(B \mid \omega) = \omega(B), \qquad \mathrm{Prob}(\neg B \mid \omega) = \omega(\mathbf{1} - B).$$

Unsharp unless B is a projection.

Because ε_σ is completely positive, but not a homomorphism in general:

- probe effects induce system effects
- even sharp probe effects typically induce unsharp system effects.

Post-selection and pre-instruments

Suppose a probe-effect B is tested when the system state is ω.

The post-selected system state, conditioned on the effect being observed, should correctly predict the probability of any system effect being observed, given that B was.

Post-selection and pre-instruments

Probability of a joint successful measurement of system effect A and probe effect B is

$$\mathrm{Prob}(A\&B) = (\omega \otimes \sigma)(\Theta(A \otimes B)) \stackrel{\text{def}}{=} (\mathfrak{I}_\sigma(B)(\omega))(A)$$

so
$$\mathrm{Prob}(A|B) = \frac{\mathrm{Prob}(A\&B)}{\mathrm{Prob}(B)} = \frac{(\mathfrak{I}_\sigma(B)(\omega))(A)}{(\mathfrak{I}_\sigma(B)(\omega))(\mathbf{1})},$$

Call $\mathfrak{I}_\sigma(B) : \mathcal{A}(\boldsymbol{M})^*_+ \to \mathcal{A}(\boldsymbol{M})^*_+$ a pre-instrument.

Post-selection and pre-instruments

Probability of a joint successful measurement of system effect A and probe effect B is

$$\mathrm{Prob}(A\&B) = (\omega\otimes\sigma)(\Theta(A\otimes B)) \stackrel{\text{def}}{=} (\mathcal{I}_\sigma(B)(\omega))(A)$$

so
$$\mathrm{Prob}(A|B) = \frac{\mathrm{Prob}(A\&B)}{\mathrm{Prob}(B)} = \frac{(\mathcal{I}_\sigma(B)(\omega))(A)}{(\mathcal{I}_\sigma(B)(\omega))(\mathbf{1})},$$

Call $\mathcal{I}_\sigma(B):\mathcal{A}(\boldsymbol{M})^*_+\to\mathcal{A}(\boldsymbol{M})^*_+$ a pre-instrument.

The post-selected state, conditioned on B, is $\boxed{\omega' = \dfrac{\mathcal{I}_\sigma(B)(\omega)}{(\mathcal{I}_\sigma(B)(\omega))(\mathbf{1})}}$ (if defined).

Non-selective measurement results in $\omega_B = \mathcal{I}_\sigma(B)(\omega) + \mathcal{I}_\sigma(\mathbf{1}-B)(\omega) = \mathcal{I}_\sigma(\mathbf{1})(\omega)$ independent of B!

Locality and post-selection

Theorem For A localisable in $K^\perp$, $$\omega'(A) = \frac{\omega(A\varepsilon_\sigma(B))}{\omega(\varepsilon_\sigma(B))}$$

Corollary $\omega'(A) = \omega(A)$ iff A is uncorrelated with $\varepsilon_\sigma(B)$ in ω.

$\omega'(A) = \omega(A)$ for nonselective measurement of B

Locality and post-selection

Theorem For A localisable in $K^\perp$, $\qquad \omega'(A) = \dfrac{\omega(A\varepsilon_\sigma(B))}{\omega(\varepsilon_\sigma(B))}$

Corollary $\omega'(A) = \omega(A)$ iff A is uncorrelated with $\varepsilon_\sigma(B)$ in ω.

$\omega'(A) = \omega(A)$ for nonselective measurement of B

If ω has a Reeh–Schlieder property, (e.g., Minkowski vacuum state)

$$\omega'(A) = \omega(A) \iff \varepsilon_\sigma(B) = \omega(\varepsilon_\sigma(B))\mathbf{1}$$

for observables A localisable in $K^\perp$.

Post-selecting on nontrivial effects alters expectation values in $K^\perp$ due to correlation.

Example – correlated observables

Commuting effects A, A' are perfectly correlated in state ω if

$$\mathrm{Prob}(A\&A') + \mathrm{Prob}(\neg A\&\neg A') = 1 \qquad \Longleftrightarrow \qquad \omega(A(\mathbf{1} - A')) = 0 = \omega((\mathbf{1} - A)A')$$

Consider a measurement scheme in which $A = \varepsilon_\sigma(B)$ for probe effect B.
If ω' is the updated state, conditioned on successful measurement of B, then

$$\mathrm{Prob}(A' \mid \omega') = \omega'(A') = 1$$

Example – correlated observables

Commuting effects A, A' are perfectly correlated in state ω if

$$\text{Prob}(A\&A') + \text{Prob}(\neg A\&\neg A') = 1 \qquad \iff \qquad \omega(A(\mathbf{1} - A')) = 0 = \omega((\mathbf{1} - A)A')$$

Consider a measurement scheme in which $A = \varepsilon_\sigma(B)$ for probe effect B.
If ω' is the updated state, conditioned on successful measurement of B, then

$$\text{Prob}(A' \mid \omega') = \omega'(A') = 1 - \mathcal{E}$$

where $\mathcal{E} \geq 0$ is bounded by

$$\mathcal{E}^2 \leq (\omega \otimes \sigma)(\Delta^2)\left(1 + \frac{\text{Var}(\widetilde{B}; \underset{\sim}{\omega}_\sigma)}{\omega(A)^2}\right), \qquad \Delta = (\mathrm{id} - \Theta)(A' \otimes \mathbf{1}).$$

Both factors can be reduced by experimental design.

Example – correlated observables

Commuting effects A, A' are perfectly correlated in state ω if

$$\mathsf{Prob}(A\&A') + \mathsf{Prob}(\neg A\&\neg A') = 1 \qquad \iff \qquad \omega(A(\mathbf{1}-A')) = 0 = \omega((\mathbf{1}-A)A')$$

Consider a measurement scheme in which $A = \varepsilon_\sigma(B)$ for probe effect B.
If ω' is the updated state, conditioned on successful measurement of B, then

$$\mathsf{Prob}(A' \mid \omega') = \omega'(A') = 1 - \mathcal{E}$$

where $\mathcal{E} \geq 0$ is bounded by

$$\mathcal{E}^2 \leq (\omega\otimes\sigma)(\Delta^2)\left(1 + \frac{\mathsf{Var}(\widetilde{B};\underset{\sim}{\omega}_\sigma)}{\omega(A)^2}\right), \qquad \Delta = (\mathrm{id}-\Theta)(A'\otimes\mathbf{1}).$$

$\mathsf{Prob}(A' \mid \omega') \to 1$ in the limit of ideal experimentation

What versus how?

The updated state depends on Θ and B; not just the system observable $A = \varepsilon_\sigma(B)$.
Depends on how the measurement was made, not just what was measured.

What versus how?

The updated state depends on Θ and B; not just the system observable $A = \varepsilon_\sigma(B)$. Depends on how the measurement was made, not just what was measured.

However, $$\mathcal{E} := \max\left\{\left|\omega'(A') - \frac{\omega(A'\varepsilon_\sigma(B))}{\omega(\varepsilon_\sigma(B))}\right|, \left|\omega'(A') - \frac{\omega(\varepsilon_\sigma(B)A')}{\omega(\varepsilon_\sigma(B))}\right|\right\},$$

obeys $$\mathcal{E}^2 \le (\omega\otimes\sigma)(\Delta^2)\left(1 + \frac{\mathrm{Var}(\widetilde{B}; \underset{\sim}{\omega}_\sigma)}{\omega(\varepsilon_\sigma(B))^2}\right), \qquad \Delta = (\mathrm{id} - \Theta)(A'\otimes\mathbf{1}).$$

$\omega'(A') \approx \dfrac{\omega(\{A', \varepsilon_\sigma(B)\})}{2\omega(\varepsilon_\sigma(B))}$ for those A' only slightly affected by the interaction.

Successive measurement of two probes

For $i = 1, 2$ consider $\mathcal{B}_i$ with coupling regions K_i and scattering morphisms Θ_i.

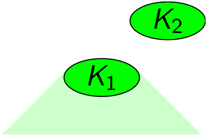
Combined probe $\mathcal{B}_1 \otimes \mathcal{B}_2$ has coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Successive measurement of two probes

For $i = 1, 2$ consider $\mathcal{B}_i$ with coupling regions K_i and scattering morphisms Θ_i.

Combined probe $\mathcal{B}_1 \otimes \mathcal{B}_2$ has coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Suppose $K_2 \cap J^-(K_1) = \emptyset$, so K_2 is later than K_1 according to some observers and assume causal factorisation, i.e.,



$$\hat{\Theta} = \hat{\Theta}_1 \circ \hat{\Theta}_2, \quad \text{where } \hat{\Theta}_1 = \Theta_1 \otimes_3 \mathrm{id} \quad \text{and } \hat{\Theta}_2 = \Theta_2 \otimes_2 \mathrm{id}$$

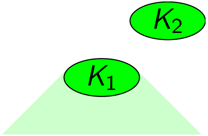
Successive measurement of two probes

For $i = 1, 2$ consider $\mathcal{B}_i$ with coupling regions K_i and scattering morphisms Θ_i.

Combined probe $\mathcal{B}_1 \otimes \mathcal{B}_2$ has coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Suppose $K_2 \cap J^-(K_1) = \emptyset$, so K_2 is later than K_1 according to some observers and assume causal factorisation, i.e.,

$$\hat{\Theta} = \hat{\Theta}_1 \circ \hat{\Theta}_2, \quad \text{where } \hat{\Theta}_1 = \Theta_1 \otimes_3 \mathrm{id} \quad \text{and } \hat{\Theta}_2 = \Theta_2 \otimes_2 \mathrm{id}$$



Theorem Coherence of successive measurement

$$\mathcal{I}_{\sigma_2}(B_2) \circ \mathcal{I}_{\sigma_1}(B_1) = \mathcal{I}_{\sigma_1 \otimes \sigma_2}(B_1 \otimes B_2)$$

Post-selection on B_1 and then B_2 agrees with post-selection on $B_1 \otimes B_2$.

Successive measurement of two probes

For $i = 1, 2$ consider $\mathcal{B}_i$ with coupling regions K_i and scattering morphisms Θ_i.

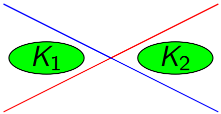
Combined probe $\mathcal{B}_1 \otimes \mathcal{B}_2$ has coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Suppose $K_2 \cap J^-(K_1) = \emptyset$, so K_2 is later than K_1 according to some observers and assume causal factorisation, i.e.,

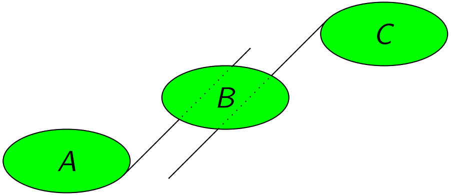
$$\hat{\Theta} = \hat{\Theta}_1 \circ \hat{\Theta}_2, \quad \text{where } \hat{\Theta}_1 = \Theta_1 \otimes_3 \mathrm{id} \quad \text{and } \hat{\Theta}_2 = \Theta_2 \otimes_2 \mathrm{id}$$

Corollary If K_1 and K_2 are causally disjoint,

$$\mathcal{I}_{\sigma_2}(B_2) \circ \mathcal{I}_{\sigma_1}(B_1) = \mathcal{I}_{\sigma_1 \otimes \sigma_2}(B_1 \otimes B_2) = \mathcal{I}_{\sigma_1}(B_1) \circ \mathcal{I}_{\sigma_2}(B_2)$$



Impossible measurements? Bostelmann, CJF & Ruep `arXiv:2003.04660`

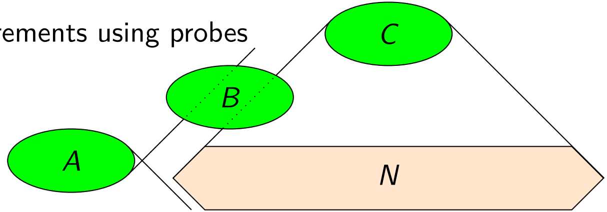


- Alice chooses whether to make a nonselective measurement
- Bob certainly makes a nonselective measurement
- Can Charlie determine whether Alice performed the measurement?

$$\omega_{AB}(C) \overset{?}{\neq} \omega_B(C)$$

Impossible measurements? Bostelmann, CJF & Ruep `arXiv:2003.04660`

Model A and B measurements using probes



More detailed investigation of scattering map locality properties gives

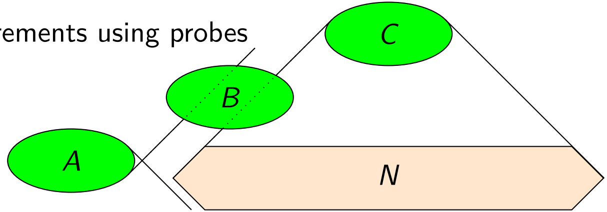
$$\hat{\Theta}_2 C \otimes \mathbf{1} \otimes \mathbf{1} \in \mathcal{U}(\boldsymbol{M}; N) \qquad \text{for region } N \subset K_A^\perp \cap M_B^-$$

Consequently, Charlie cannot determine whether Alice has measured:

$$\omega_{AB}(C) = (\omega \otimes \sigma_1 \otimes \sigma_2)(\hat{\Theta}_1 \hat{\Theta}_2 C \otimes \mathbf{1} \otimes \mathbf{1}) = (\omega \otimes \sigma_1 \otimes \sigma_2)(\hat{\Theta}_2 C \otimes \mathbf{1} \otimes \mathbf{1}) = \omega_B(C)$$

Impossible measurements? Bostelmann, CJF & Ruep `arXiv:2003.04660`

Model A and B measurements using probes



The analysis shows that the measurement scheme is free of Sorkin-type pathologies.

Key assumption – the probes and couplings are described by physics respecting locality.

Impossible measurements can only be performed using impossible apparatus.

A specific probe model

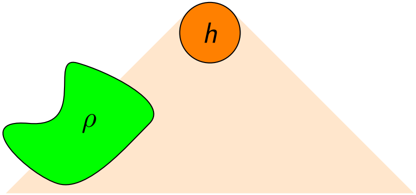
Two free scalar fields: Φ (system) and Ψ (probe) coupled via an interaction term

$$\mathcal{L}_{\text{int}} = -\rho\Phi\Psi, \qquad \rho \in C_0^\infty(M), \qquad K = \operatorname{supp}\rho.$$

Linear equations: standard quantisation applies at least for sufficiently weak coupling.
As formal power series in $h \in C_0^\infty(M^+)$,

$$\Theta(\mathbf{1} \otimes e^{i\Psi(h)}) = e^{i\Phi(f^-)} \otimes e^{i\Psi(h^-)}$$

where f^- and $h^- - h$ are supported in $\operatorname{supp}\rho \cap J^-(\operatorname{supp} h)$.



A specific probe model

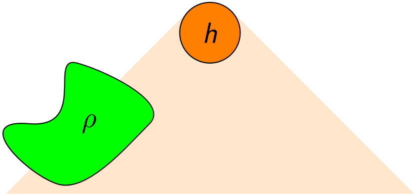
Two free scalar fields: Φ (system) and Ψ (probe) coupled via an interaction term

$$\mathcal{L}_{\text{int}} = -\rho\Phi\Psi, \qquad \rho \in C_0^\infty(M), \qquad K = \operatorname{supp}\rho.$$

Linear equations: standard quantisation applies at least for sufficiently weak coupling. As formal power series in $h \in C_0^\infty(M^+)$,

$$\Theta(\mathbf{1} \otimes e^{i\Psi(h)}) = e^{i\Phi(f^-)} \otimes e^{i\Psi(h^-)}$$

where f^- and $h^- - h$ are supported in $\operatorname{supp}\rho \cap J^-(\operatorname{supp} h)$.



$$\varepsilon_\sigma(e^{i\Psi(h)}) = \sigma\left(e^{i\Psi(h^-)}\right) e^{i\Phi(f^-)} = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

if σ is quasifree with two-point function S.

Examples of induced observables

$$\varepsilon_\sigma(e^{i\Psi(h)}) = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

$$\begin{aligned} \varepsilon_\sigma(\Psi(h)) &= \Phi(f^-) \\ \varepsilon_\sigma(\Psi(h)^2) &= \Phi(f^-)^2 + S(h^-, h^-)\mathbf{1} \end{aligned}$$

Consequently,

$$\begin{aligned} \mathbb{E}(\widetilde{\Psi(h)}; \underset{\sim}{\omega}_\sigma) &= \omega(\Phi(f^-)) \\ \mathrm{Var}(\widetilde{\Psi(h)}; \underset{\sim}{\omega}_\sigma) &= \mathrm{Var}(\Phi(f^-); \omega) + S(h^-, h^-) \end{aligned}$$

Increased variance in true measurement from detector fluctuations.

Deformed product on the probe system

$\varepsilon_\sigma : \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ is not a homomorphism. BUT $\exists$ a deformed product on $\mathcal{B}(\boldsymbol{M})$,

$$e^{i\Psi(h)} \star e^{i\Psi(h')} = \frac{\sigma(e^{i\Psi(h)})\sigma(e^{i\Psi(h')})}{\sigma(e^{i\Psi(h+h')})} e^{-iE_P(f^-, f'^-)/2} e^{i\Psi(h+h')}$$

in which ε_σ is a homomorphism (though not injective).
Consequence: the induced observables form a subalgebra of $\mathcal{A}(\boldsymbol{M})$

$$\mathrm{Im}\, \varepsilon_\sigma \cong \mathcal{B}(\boldsymbol{M}) / \ker \varepsilon_\sigma .$$

so the system is partially represented in the probe algebra.

Example: $\Psi(h)$'s do not necessarily $\star$-commute at spacelike separation,

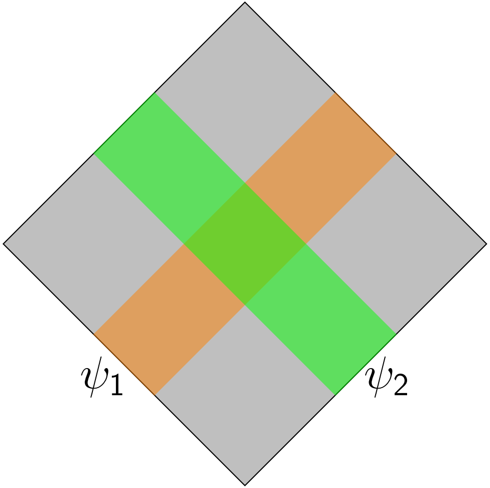
$$[\Psi(h), \Psi(h')]_\star = iE_P(f^-, f'^-),$$

allowing for the creation of long-range correlations.

Summary

- Operational framework of QMT adapted to AQFT
  - covariant, formulated for curved as well as flat spacetimes
  - framework derived from minimal assumptions
- Probe observables induce local system observables,
  - localisable in the causal hull of coupling region
- Post-selected states
  - updated state derived from required properties rather than posited
  - reproduces idealised correlations in a limit of idealised measurement
  - coherence under successive measurements
  - invariant under re-ordering of causally disjoint measurements
- Framework is free of impossible measurements
- Computation of induced observables for specific model

Local modification of couplings in QFT



An interaction term

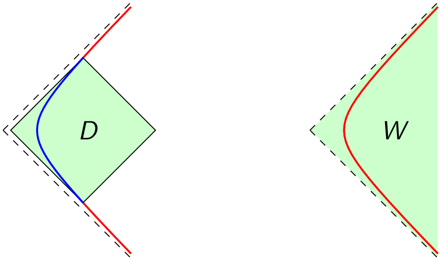
$$\psi_1\psi_2\varphi_1\varphi_2$$

provides a tunable coupling between φ_1 and φ_2.

Localisation of induced observables

$\varepsilon_\sigma(\Psi(h)^n)$ may be localised in any open causally convex nhd of

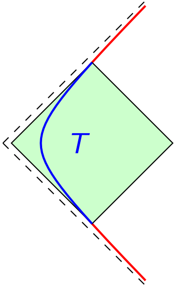
$$\mathrm{supp}\, f^- \subset \mathrm{supp}\,\rho \cap J^-(\mathrm{supp}\, h)$$

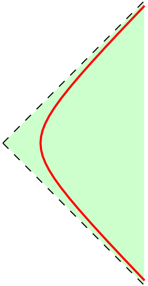


Localisation region for finite-time coupling is a diamond D.

Localisation region for eternal coupling is a wedge W (can't do better).

Back to Unruh





Localisation region for finite-time coupling is a diamond D
Measurements may be taken along future of curve beyond D.
Localisation region for eternal coupling is a wedge. HIghly nonlocal.