INTRODUCTION TO NONCOMMUTATIVE PROBABILITY THEORY

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In my talk I will give an introduction to noncommutative probability, a generalization of classical probability theory to the context of von Neumann algebras. Specifically, a noncommutative probability space consists of a pair $(\mathcal{M}, \tau)$, where $\mathcal{M}$ is a von Neumann algebra and τ is a (normal, faithful) state or trace on $\mathcal{M}$. The elements of $\mathcal{M}$ are interpreted as random variables, the projections in $\mathcal{M}$ play the role of events and τ takes the role of the expectation functional.

I will explain how classical probability theory fits into this framework and give some other natural examples of noncommutative probability spaces. Many notions from classical probability have a natural generalization in the noncommutative setting (e.g. conditional expectation, almost sure convergence), but other ones, such as stopping times and independence, are more intricate. I will discuss some of these notions, paying special attention to independence, for which several interesting alternatives exist in the noncommutative context. In the final part of the talk I will survey some recent results on generalizations of probabilistic L^p-inequalities (such as Khintchine's inequality). If time permits, I will discuss some results of my own making in this area.

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